

## Response to D.T. Son's comment on "Is there a 'most perfect fluid' consistent with quantum field theory?"

D.T. Son raises an extremely interesting and subtle point in his comment[1]. However, the conclusion in the original letter that theoretically consistent exceptions exist for the proposed general bound that  $\eta/s \geq (4\pi)^{-1}$  for all fluids [2], appears to remain unaffected by the issue raised.

Firstly, in the context of nonrelativistic quantum mechanics there exist theoretically consistent systems which violate the putative bound [3] and are unaffected by the issue raised in the comment. Thus, the origin of any general bound must lie beyond quantum mechanics. One suggestion is that relativistic quantum field theories with sensible behavior in the ultraviolet somehow act as a censor by preventing *any* non-relativistic system arising from such a field theory to violate the bound. It was pointed out in ref. [3] that the ultraviolet behavior of many-flavored QCD acts this way in preventing a pion gas from violating the bound. However, it appears unlikely *a priori* that the bound is fundamentally tied to relativistic field theoretic effects, if for no other reason than the fact, first noted in Ref. [2], that  $c$  is absent from the bound. Thus, there are deep reasons to doubt a general bound regardless of the issue raised in [1].

It was suggested in Ref. [3] that if a bound on  $\eta/s$  is general, it is natural to expect it to hold (up to possible small violations due to ambiguities) for metastable fluids provided that the fluid is in a sufficiently well-defined macroscopic state that the entropy is essentially well defined, and that  $\eta$  is essentially well defined in that the characteristic smallest time-scale for fluid behavior is much smaller than the characteristic decay time. The heavy meson gas system considered in Ref. [3] *is* in this class and *does* violate the bound. Thus, regardless of the validity of the issue raised in Ref. [1], Ref. [3] at a minimum establishes the existence of a theoretically consistent example that demonstrates a limitation of the class of systems for which such a general bound can hold.

The critique in Ref. [1] is not aimed at the validity of the heavy meson gas of Ref. [3] as a counterexample to the bound *per se*, but rather at the expectation that the bound should apply to systems in that class. The key insight is that for the bound to hold, it might not be sufficient for both  $\eta$  and  $s$  to be essentially well defined on their own terms (hydrodynamic and thermodynamic, respectively), but may require the system to live long enough so one can *simultaneously* measure both  $\eta$  and  $s$ . Given the connection between  $\eta$  and  $s$  in the bound, it is not unreasonable that its application be limited to systems in which  $\eta$  and  $s$  are simultaneously well-defined. However, this restriction does not invalidate a heavy meson system as a counterexample to the bound.

Reference [1] argues that in order to simultaneously measure  $\eta$  and  $s$  for the heavy meson system of Ref. [3],  $\eta$  must be measured over a thermodynamic length scale associated with the size of the minimum system for which  $s$  is well defined (which scales as  $\exp(\xi^4/3)$ ) rather than the hydrodynamic scale (which is a power law in  $\xi$ ). While  $\eta$  is essentially well defined on the hydrodynamic scale, it need not be over the much larger thermodynamic scale, since the metastable fluid presumably decays before a measurement over this scale is complete. Thus it is argued that the heavy meson system does not have  $\eta$  and  $s$  simultaneously well defined.

However, it is *not* necessary to measure  $\eta$  over the thermodynamic length scale identified in [1] to have  $\eta$  and  $s$  essentially well defined at the same time. The thermodynamic limit is determined by a system's *volume* and not its length scale (provided that all lengths of the system are large compared to the thermal wavelength—as is the case for the heavy meson system in Ref. [3]). Consider, for example, the stereotypical setup for measuring the viscosity: fluid is contained between two parallel rectangular plates whose cross-sectional size is  $A$  and whose separation is  $d$  with  $d \ll A^{1/2}$ ;  $\eta$  is determined by the force needed to keep one plate moving with fixed velocity relative to the other. For the heavy meson system of Ref. [3] this setup means that the system is in the thermodynamic limit in the sense that  $s$  is essentially well defined if  $Ad \gg N_f \sim \exp(\xi^4)$ ;  $\eta$  is essentially well defined provided that  $d$  is much larger than the mean-free path,  $l$ , ( $l \sim \xi^4$ ) while being much smaller than the scale characterizing decay (which scales as a power law in  $\xi$  times  $l$  [4]). Thus, if  $A = \alpha \exp(\xi^4)$  and  $d = \beta \xi^4$  with  $\alpha$  and  $\beta$  sufficiently large constants, the system at large  $\xi$  will violate the proposed bound with  $\eta$  and  $s$  determined simultaneously and each essentially well defined.

Numerous insightful comments of D.T. Son, A. Cherman and P. Hohler are gratefully acknowledged. This work was supported by the U.S. Department of Energy.

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